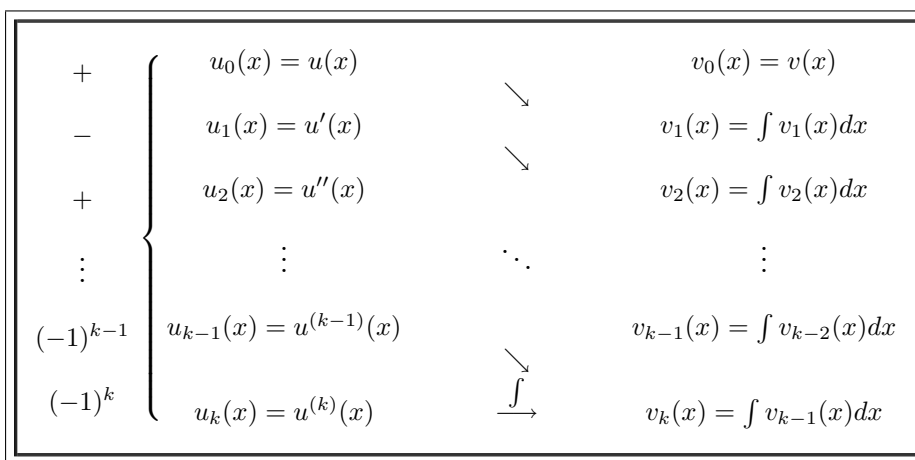


Integration by Parts

Consider the following integral:

$$I = \int u(x)v(x) dx.$$

By differentiating $u(x)$ and integrating $v(x)$ as many times as needed, we may find I by using the following chart.



Thus

$$I = \int u(x)v(x) dx = \sum_{i=0}^{k-1} (-1)^i u_i(x)v_{i+1}(x) + (-1)^k \int u_k(x)v_k(x) dx.$$

♣ **Examples**

$I = \int x^3 e^{2x} dx$	$J = \int e^{2x} \sin x dx$
$+ \begin{cases} u_0(x) = x^3 \\ - u_1(x) = 3x^2 \\ + u_2(x) = 6x \\ - u_3(x) = 6 \\ + u_4(x) = 0 \end{cases} \begin{matrix} \searrow \\ \searrow \\ \searrow \\ \searrow \\ \xrightarrow{\int} \end{matrix} \begin{matrix} v_0(x) = e^{2x} \\ v_1(x) = \frac{1}{2}e^{2x} \\ v_2(x) = \frac{1}{4}e^{2x} \\ v_3(x) = \frac{1}{8}e^{2x} \\ v_4(x) = \frac{1}{16}e^{2x} \end{matrix}$	$+ \begin{cases} u_0(x) = e^{2x} \\ - u_1(x) = 2e^{2x} \\ + u_2(x) = 4e^{2x} \end{cases} \begin{matrix} \searrow \\ \searrow \\ \xrightarrow{\int} \end{matrix} \begin{matrix} v_0(x) = \sin x \\ v_1(x) = -\cos x \\ v_2(x) = -\sin x \end{matrix}$
$I = e^{2x} \left[\frac{x^3}{2} - \frac{3x^2}{4} + \frac{3x}{4} - \frac{3}{8} \right] + C$	$J = e^{2x} [-\cos x + 2 \sin x] - 4J$ $J = \frac{1}{5} [-\cos x + 2 \sin x] + C$

The following integrals involve integration by parts, where $v_0(x) = 1$:

$$\int \ln x dx \quad \int \sin^{-1} x dx \quad \int \cos^{-1} x dx \quad \int \tan^{-1} x dx \quad \int \cot^{-1} x dx \quad \int \sec^{-1} x dx \quad \int \csc^{-1} x dx$$